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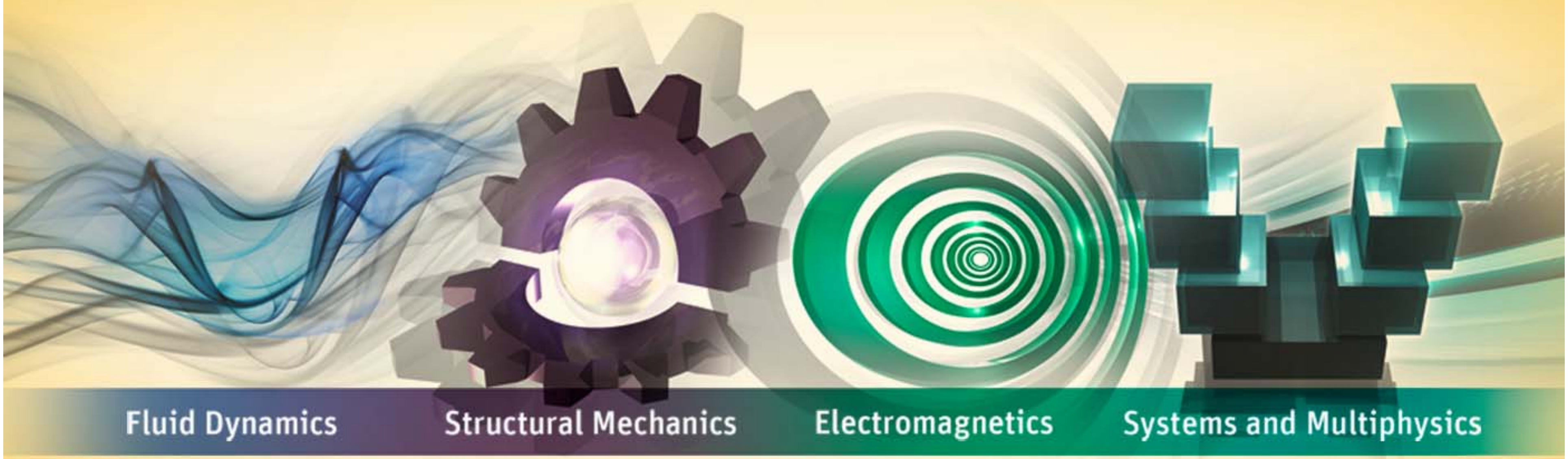
Multiple Wave Spectra



Richard May – Team Lead, Aqwa development

1. Introduction

Ocean waves with different frequencies and directions
are very difficult to model mathematically

Various simplified theories & spectral models of ocean waves

- small amplitude linear Airy wave
- higher order Stokes wave
- long crested irregular waves
- multiple directional (short crested) irregular waves

Waves induce forces on offshore structures:

- the wave exciting forces at wave frequency
- non-linear wave forces:

  low frequency drift force & sum frequency forces,

  due to the instantaneous wetted hull surface varying,

  impact, slamming forces.

Long crested wave case does not necessarily lead to conservative results,

the interaction effect between waves from different directions may be important (Renaud et al. 2008)

2. Ocean Waves Modeling in Aqwa

13.0 and before	14.0 and thereafter
• Airy wave • Second order Stokes wave	same
• Formulated spectrum • User define spectrum • Imported wave elevation (IWHT)	same
• Main + cross swell	same
• Spreading waves • 2-D carpet waves	same

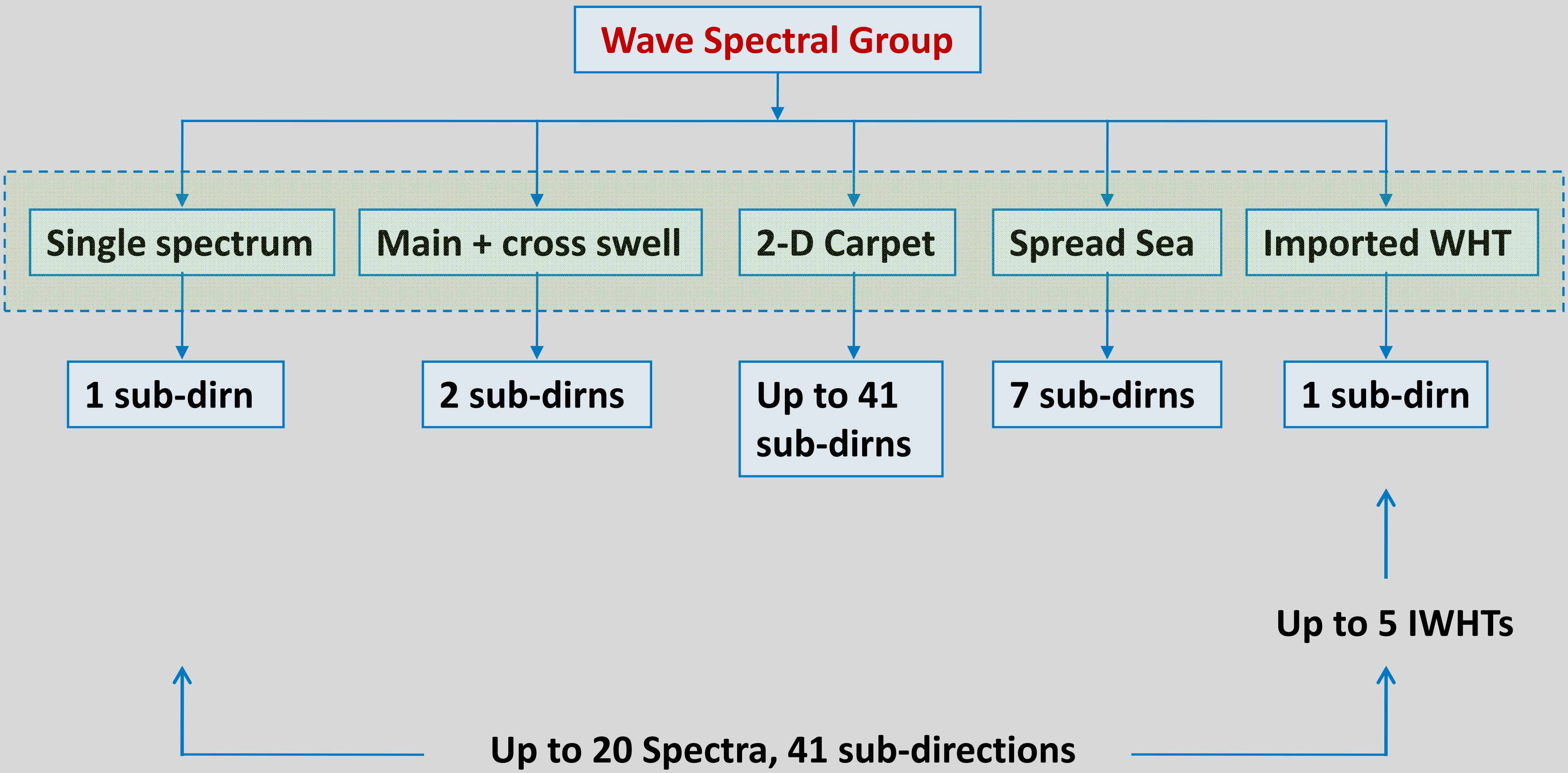
Spectral group

- Any combination

14.0 - in Librium and Drift
14.5 - in Fer

one sub-dirn in Line & Naut

Structure of wave spectral group



* IWHTs: Imported files of wave elevation time history

Definition of spectral group in Aqwa-suite

```
    13SPGR
    13SGNM          UDEF + PSMZ + 2 IWHTs
    13CURR                 0.5       50.0
    13ISOW
    13WIND                15.0      150.0
*--------------------------------------------------
    13NAME          1.UDEF - LONG CRESTED
*--------------------------------------------------
    13SEED 1005
    13SPDN               0.000
    13UDEF              0.3142     0.0000
      ...
    13UDEF              1.8850     0.4618
    13FINI
*---------------------------------------------------
    13NAME          2. LONG CRESTED -FORMULATTED
*---------------------------------------------------
    13SEED 1006
    13PSMZ                 0.3        2.5     0.001      7.0
*---------------------------------------------------
    13NAME          3. IMPORTED WAVE ELEVATION
*---------------------------------------------------
    13SEED 1007
    13IWHT          MTWAVE1.WHT
    13IWHT          MTWAVE2.WHT
    13NODR
 END13
```

Spectral group & name

One current/wind setting

Wave spectrum definition

Seed definition
(see Aqwa reference 4.13.6 for default values)

Ignore 2nd force of this sub-directional spectrum

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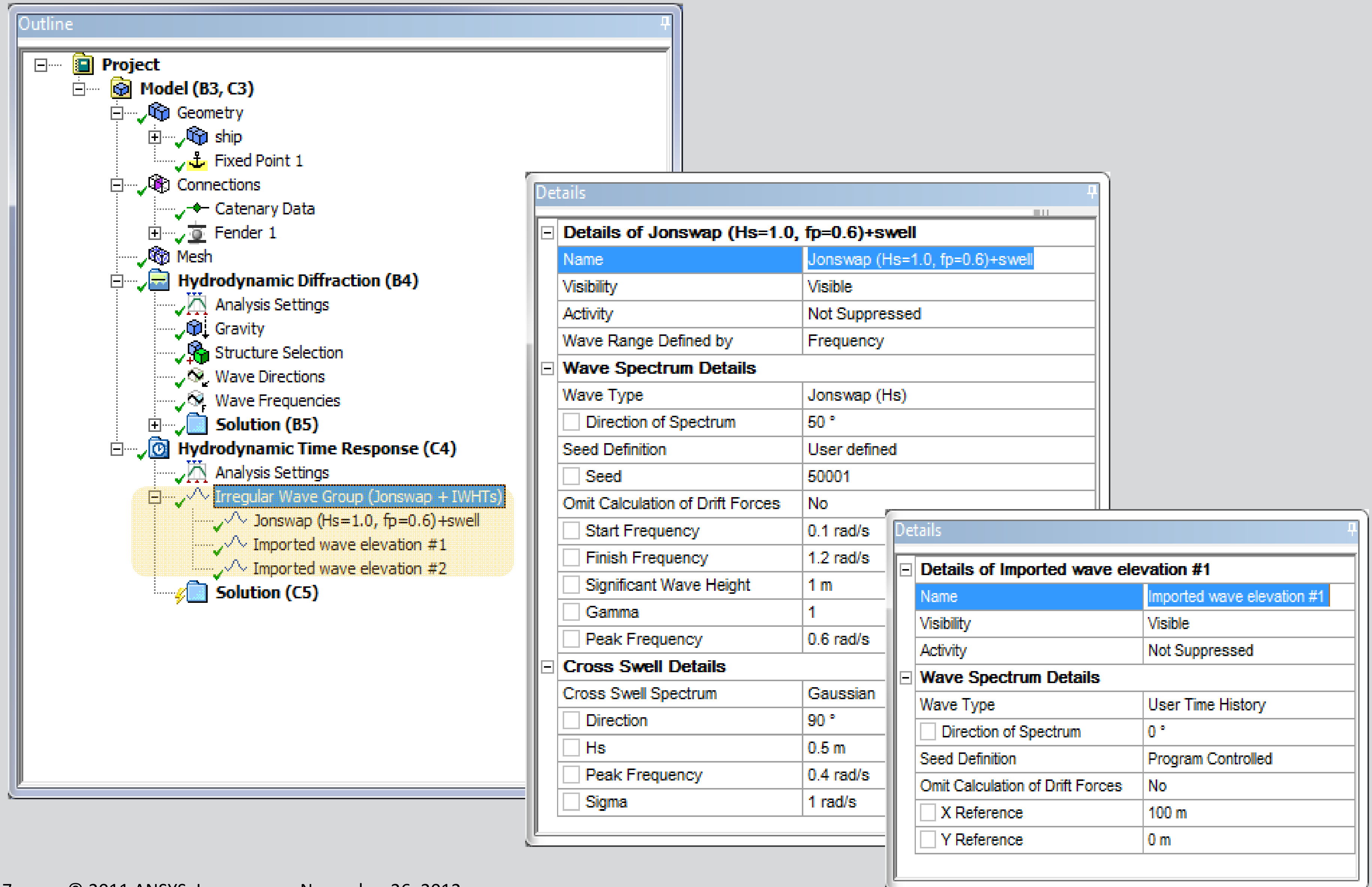
Definition of spectral group in Aqwa-Workbench



3. Multi-Directional Wave Effects on Loads

- **Wave representation:** **Linear superposition – no interaction**

$$\zeta(\vec{X},t) = \sum_{m=1}^{N_d} \sum_{j=1}^{N_w} a_{jm} e^{i(K_{jm} X \cos\theta_m + K_{jm} Y \sin\theta_m - \omega_{jm} t + \varepsilon_{jm})}$$

- **1st order wave exciting force:** **Linear superposition**

$$\vec{F}^{(1)}(t) = \sum_{m=1}^{N_d} \sum_{j=1}^{N_w} a_{jm} \vec{F}_{jm}^{(1)} e^{i(-\omega_{jm} t + \varepsilon_{jm})}$$

- **Morison drag force:** **Linear superposition of fluid particle velocities**

$$F_d(t) = -\frac{1}{2} \rho C_d \left| U_r(t) \right| U_r(t),$$

$$U_r(t) = \sum_{m=1}^{N_d} \sum_{j=1}^{N_w} a_{jm} V_{jm}(\vec{X}) e^{i(-\omega_{jm} t + \varepsilon_{jm})} + U_C - U_S$$

ANSYS

- **Second order wave forces**

$$\vec{F}^{(2)}(t)=\sum_{m=1}^{N_d}\sum_{n=1}^{N_d}\sum_{j=1}^{N_m}\sum_{k=1}^{N_n} a_{jm}a_{kn}\{\ \vec{P}^{+}_{jkmn}\cos[(\omega_{jm}+\omega_{kn})t-(\varepsilon_{jm}+\varepsilon_{kn})]$$
$$+\vec{Q}^{+}_{jkmn}\sin[(\omega_{jm}+\omega_{kn})t-(\varepsilon_{jm}+\varepsilon_{kn})]$$
$$+\vec{P}^{-}_{jkmn}\cos[(\omega_{jm}-\omega_{kn})t-(\varepsilon_{jm}-\varepsilon_{kn})]$$
$$+\vec{Q}^{-}_{jkmn}\sin[(\omega_{jm}-\omega_{kn})t-(\varepsilon_{jm}-\varepsilon_{kn})]\ \},$$

➡ **Quadruple summations**

- Interaction between frequencies (sum and difference) and between directions

- **Drift damping** (Kim et al, 1997)

$$B_{11}=\sum_{m=1}^{N_d}\sum_{n=1}^{N_d}\sum_{j=1}^{N_w}\Xi_{11}(\omega_j;\beta_m,\beta_n)\bar{F}_x(\omega_j;\beta_m,\beta_n),$$

$$\Xi_{11}(\omega_j;\beta_m,\beta_n)=\frac{1}{2}\{\ (\cos\beta_m+\cos\beta_n)\cdot(k_j\frac{\partial}{\partial\omega_j}+\frac{2K_j}{C_{gj}})-\frac{1}{C_{gj}}(\sin\beta_m\frac{\partial}{\partial\beta_m}+\sin\beta_n\frac{\partial}{\partial\beta_n})\}$$

- **Mean Drift force for multiple directional wave case**

$$\bar{F}^{(2)} = \sum_{m=1}^{N_d} \sum_{n=1}^{N_d} \sum_{j=1}^{N_w} a_{jm} a_{jn} \{ P^{-}_{jjmn} \cos(\varepsilon_{jm} - \varepsilon_{jn}) - Q^{-}_{jjmn} \sin(\varepsilon_{jm} - \varepsilon_{jn}) \}$$

- **Mean Drift force for single directional wave case**

$$\bar{F}^{(2)} = \sum_{j=1}^{N_w} a_j \, P^{-}_{jj}$$

➡

- **Triple summations including directional coupling,**
- **In phase and out of phase components,**
- **Sensitive to wavelet random phases.**

Force spectrum in Fer

- **First order wave exciting force**

$$S_{F^{(1)}}(\omega) = \sum_{m=1}^{N_d} \left|F_m^{(1)}(\omega)\right|^2 S_m(\omega)$$

- **Difference frequency second order wave force** (v14.5)

$$S_{F^{(2)}}(\omega) = \sum_{m=1}^{N_d} \sum_{n=1}^{N_d} \{8\int S_m(\mu) S_n(\mu+\omega) \left|D_{mn}(\mu,\ \mu+\omega)\right|^2 d\mu\},$$

$$\left|D_{mn}(\mu,\ \mu+\omega)\right|^2 = \left|P_{mn}^{-}(\mu,\ \mu+\omega)\right|^2 + \left|Q_{mn}^{-}(\mu,\ \mu+\omega)\right|^2$$

- **Total wave force**

$$S_F(\omega) = S_{F^{(1)}}(\omega) + S_{F^{(2)}}(\omega)$$

Second order force coefficients

$$\vec{F}^{(2)} = -\frac{1}{2}\rho g \oint_{WL} \zeta_r^{(1)} \cdot \zeta_r^{(1)} \vec{n}\, dl$$ **Water line integral**

$$+ \frac{1}{2}\rho \iint_{S_0} [\nabla\Phi^{(1)} \cdot \nabla\Phi^{(1)}]\vec{n}\, dS$$ **Bernoulli**

$$+ \rho \iint_{S_0} [\vec{X}^{(1)} \cdot \nabla \frac{d\Phi^{(1)}}{dt}]\vec{n}\, dS$$ **Acceleration**

$$+ \vec{\alpha}^{(1)} \times \vec{F}^{(1)}$$ **Momentum**

$$+ \rho \iint_{S_0} \frac{d\Phi^{(2)}}{dt}\vec{n}\, dS$$ **2nd order potential**

without the 5th term and using complex values for unit wave amplitude

$$(\vec{P}^{+}_{jkmn}, \vec{Q}^{+}_{jkmn}) = -\frac{1}{4}\rho g \oint_{WL} \zeta'_{rjm} \cdot \zeta'_{rkn}\vec{n}\, dl + \frac{1}{4}\rho \iint_{S_0} [\nabla\Phi'_{jm} \cdot \nabla\Phi'_{kn}]\vec{n}\, dS$$

$$+ \frac{1}{2}\rho \iint_{S_0} [\vec{X}'_{jm} \cdot \nabla \frac{d\Phi'_{kn}}{dt}]\vec{n}\, dS + \frac{1}{2}\vec{\alpha}'_{jm} \times [\mathbf{M}_s \cdot \ddot{\vec{X}}'_{gkn}],$$

$$(\vec{P}^{-}_{jkmn}, \vec{Q}^{-}_{jkmn}) = -\frac{1}{4}\rho g \oint_{WL} \zeta'_{rjm} \cdot \zeta'^{*}_{rkn}\vec{n}\, dl + \frac{1}{4}\rho \iint_{S_0} [\nabla\Phi'_{jm} \cdot \nabla\Phi'^{*}_{kn}]\vec{n}\, dS$$

$$+ \frac{1}{2}\rho \iint_{S_0} [\vec{X}'_{jm} \cdot \nabla \frac{d\Phi'^{*}_{kn}}{dt}]\vec{n}\, dS + \frac{1}{2}\vec{\alpha}'_{jm} \times [\mathbf{M}_s \cdot \ddot{\vec{X}}'^{*}_{gkn}].$$

4. Extended Newman's Approximation

- **Store** $(\vec{P}^{+}_{jkmn}, \vec{Q}^{+}_{jkmn}, \vec{P}^{-}_{jkmn}, \vec{Q}^{-}_{jkmn})$ **database,**
- **Interpolating** $(\vec{P}^{+}_{jkmn}, \vec{Q}^{+}_{jkmn}, \vec{P}^{-}_{jkmn}, \vec{Q}^{-}_{jkmn})$ **of all wavelets at each time step,**
- **Quadruple summation of all 2nd order force components**

is numerically prohibitive

- **Newman's approximation for single directional waves**

$$\vec{P}^{-'}_{jk} = \frac{1}{2}[\vec{P}^{-}_{jj} + \vec{P}^{-}_{kk}]$$

- **Extended Newman's approximation for multiple directional waves**

$$\vec{P}^{-'}_{jkmn} = \frac{1}{2}[\vec{P}^{-}_{jjmn} + \vec{P}^{-}_{kknm}],$$

$$\vec{Q}^{-'}_{jkmn} = \frac{1}{2}[\vec{Q}^{-}_{jjmn} - \vec{Q}^{-}_{kknm}].$$

4. Newman's Approximation

ω = 1 2 .... j

ω
1
2
.
.
.
j

$$\begin{bmatrix} P & & & & \\ & P & \cdots & \cdots & X \\ & & P & & \vdots \\ & & & P & \vdots \\ & & & & P \end{bmatrix}$$

4. Extended Newman's Approximation

ω = 1 2 .... j

ω: 1, 2, ., ., ., j

$$\begin{bmatrix} \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} & & & & \\ & \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} & \cdots & \cdots & \begin{bmatrix} x_{11} & x_{12} \\ x_{21} & x_{22} \end{bmatrix} \\ & & \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} & & \vdots \\ & & & \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} & \vdots \\ & & & & \begin{bmatrix} d_{11} & d_{12} \\ d_{21} & d_{22} \end{bmatrix} \end{bmatrix}$$

- **Employing extended Newman's approximation**

$$\vec{F}^{(2)} = \sum_{m=1}^{N_d} \{ (\sum_{j=1}^{N_m} c_{jm}) \times [\sum_{n=1}^{N_d} \sum_{k=1}^{N_n} (c_{kn} \vec{P}^{-}_{kknm} - s_{kn} \vec{Q}^{-}_{kknm})] \}$$

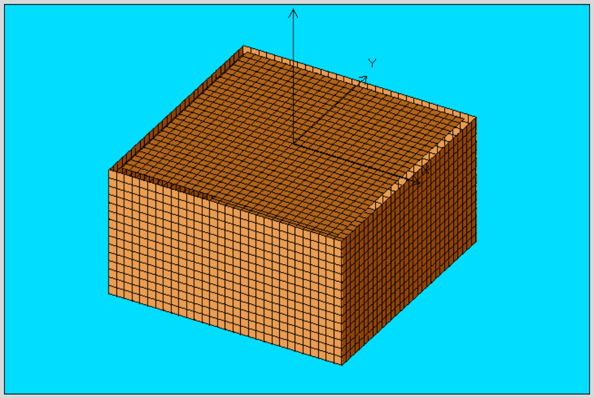
$$+ \sum_{m=1}^{N_d} \{ (\sum_{j=1}^{N_m} s_{jm}) \times [\sum_{n=1}^{N_d} \sum_{k=1}^{N_n} (s_{kn} \vec{P}^{-}_{kknm} + c_{kn} \vec{Q}^{-}_{kknm})] \}$$

where $c_{jm} = a_{jm} \cos(\omega_{jm} t - \varepsilon_{jm}),\ s_{jm} = a_{jm} \sin(\omega_{jm} t - \varepsilon_{jm})$

- **Require directional coupling mean drift force coefficients database,**
- **Obtain $(\vec{P}^{-}_{kknm}, \vec{Q}^{-}_{kknm})$ of actual relative direction at each time step,**
- **Quadruple summation reduced to triple summation.**

Less hard disk and memory requirement,
more efficient

Validation of extended Newman's approximation

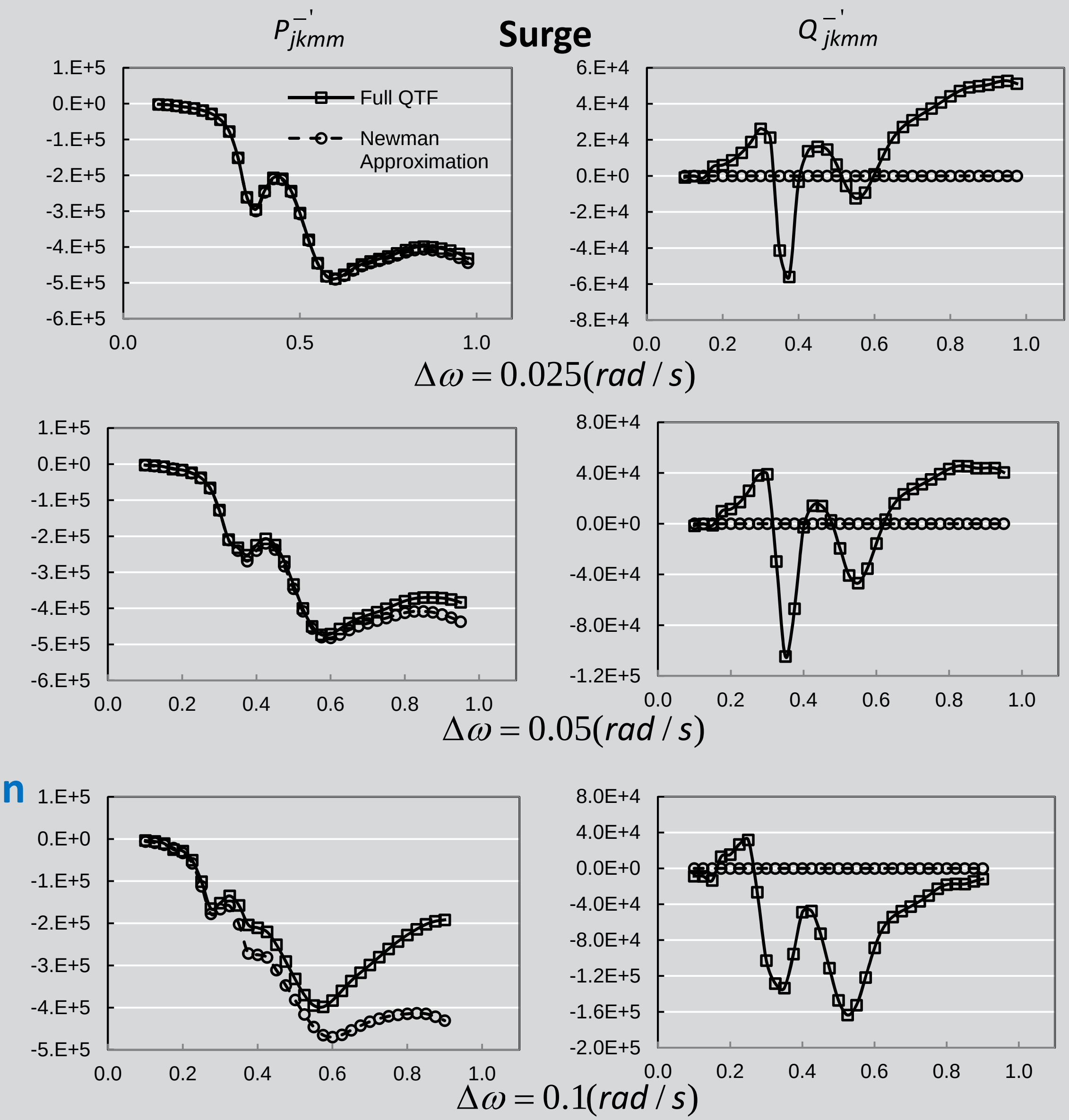


Rectangular box in deep water,
Internal lid,
No 5th term

m=n: Newman's approximation

$$\theta_m = \theta_n = -180^0$$

- $\left|P^{-}_{jkmm}\right| > \left|Q^{-}_{jkmm}\right|$
- Good for small $\Delta\omega$



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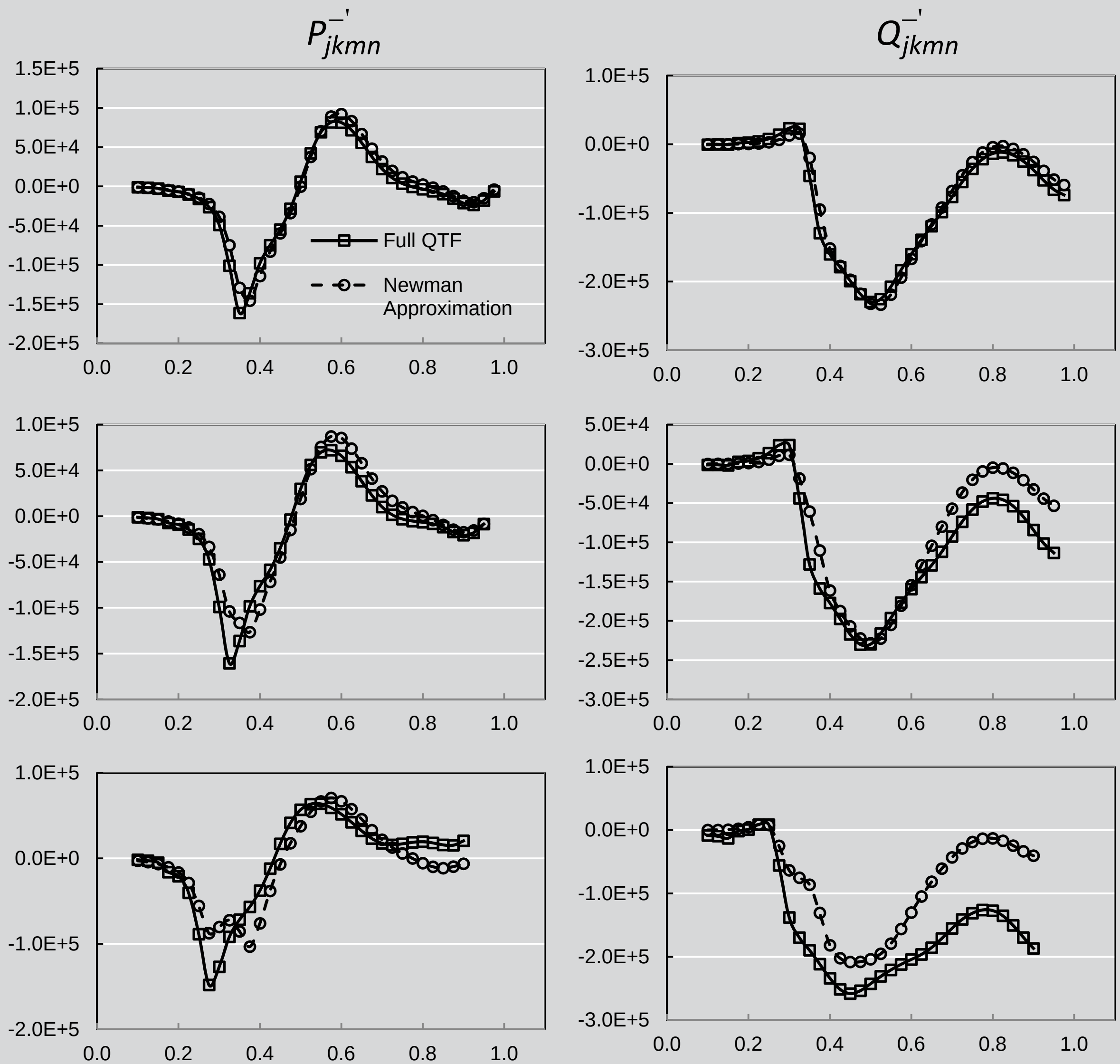
2nd order surge force $(\theta_m = -180^0,\ \theta_n = -90^0)$



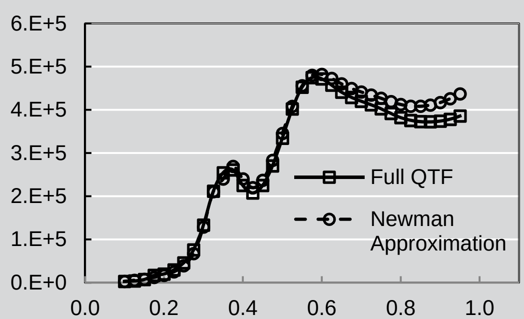
$\Delta\omega = 0.025(rad/s)$

$\Delta\omega = 0.05(rad/s)$

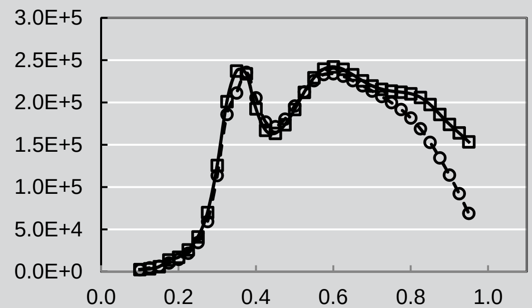
$\Delta\omega = 0.1(rad/s)$

- $|P^{-'}_{jkmn}|, |Q^{-'}_{jkmn}|$ same order
- Good for small $\Delta\omega$

Directional coupling effect on 2nd order surge force



$\Delta\theta = 0^0$

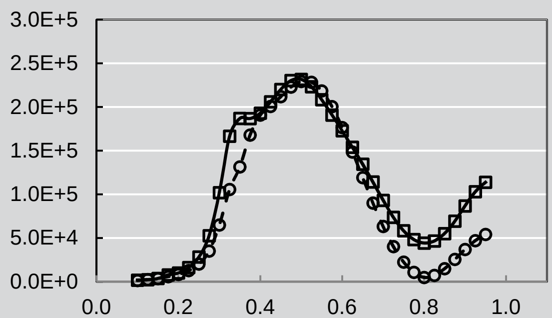


$\Delta\theta = 40^0$

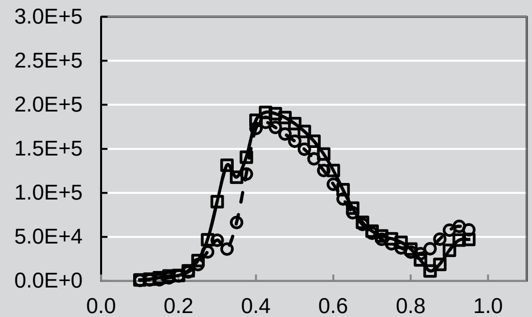
$$\sqrt{(P^{-'}_{jkmn})^2 + (Q^{-'}_{jkmn})^2}$$

$$\theta_m = -180^0$$

$$\Delta\omega = 0.05(rad/s)$$



$\Delta\theta = 90^0$

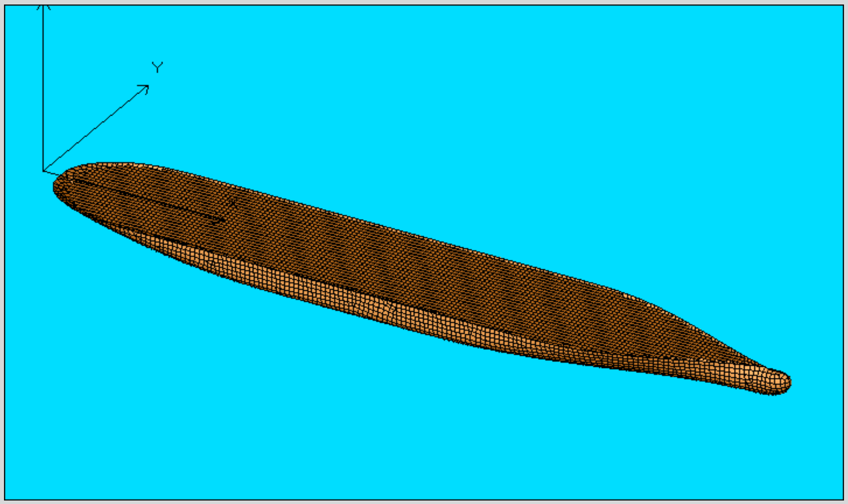


$\Delta\theta = 130^0$

Extended Newman's approximation

- **Enable to estimate** $(\vec{P}^{-}_{jkmn}, \vec{Q}^{-}_{jkmn})$ **to include directional coupling effect**
- **As good as the original Newman's approximation for multiple directions**
- **Easy accomplishment**

5. Effect of multi directional spectrum on a moored LNG carrier



13500m^3 storage capacity LNG carrier
(by courtesy of SBM)

Length between perpendiculars: 274m
Draft: 11m

Soft mooring system:
- Longitudinal mooring stiffness: 281kN/m
- Transverse mooring stiffness: 254kN/m
- Rotational mooring stiffness: 4.58E6kNm

Roll damping: 4.0e5 kNms/rad

Water depth: 15m

Spread sea:

JONSWAP spectrum: Hs=1m, Tp=10s, $\gamma = 3.3$
spreading form: $\cos^2(\theta - \overline{\theta}),\ \overline{\theta} = 180^0$

  November 26, 2012

Aqwa-Line Calculation

- **Soft spring system is defined by additional structural stiffness (SSTF)**
- **39 frequencies and 37 directions in [-180, 180] degrees**
- **Use MQTF option to calculate the directional coupling mean QTF matrices**
- **25% extra CPU time for the directional coupling mean QTF calculation**
- ***.MQT size of 12mb compared to 2.3mb of *.RES**

Effects on equilibrium position

Three treatments of spreading sea:

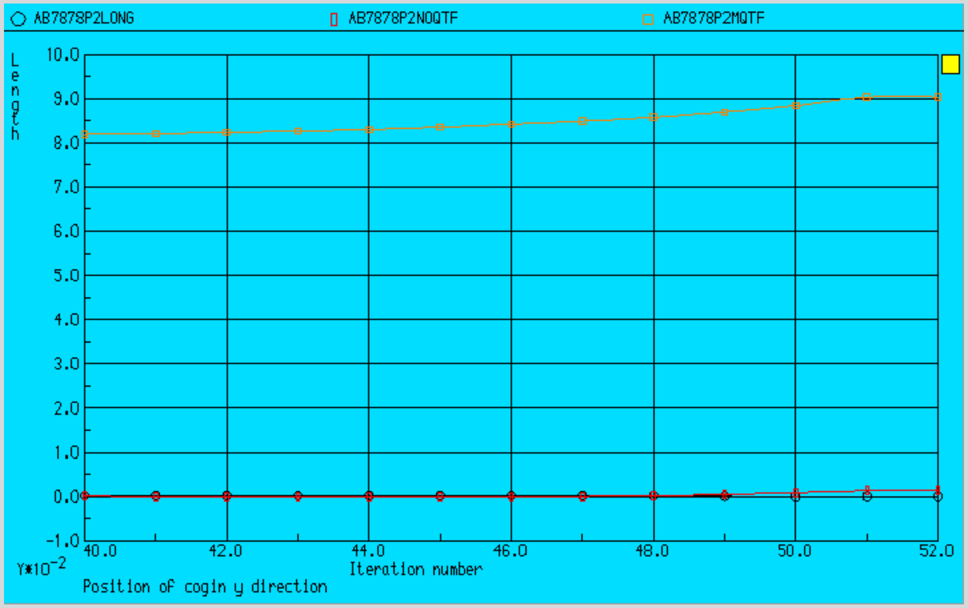
(1) Long crested wave along main heading – AB7878P2LONG

(2) Represented by 7-point Gaussian integration, no directional coupling MQTF - AB7878P2NOQTF

(3) Represented by 7-point Gaussian integration, with directional coupling MQTF – AB7878P2MQTF

```
    91    DRM1
    91FILE          AL7878P2M15.MQT
    91CSTR   1
 END91
```

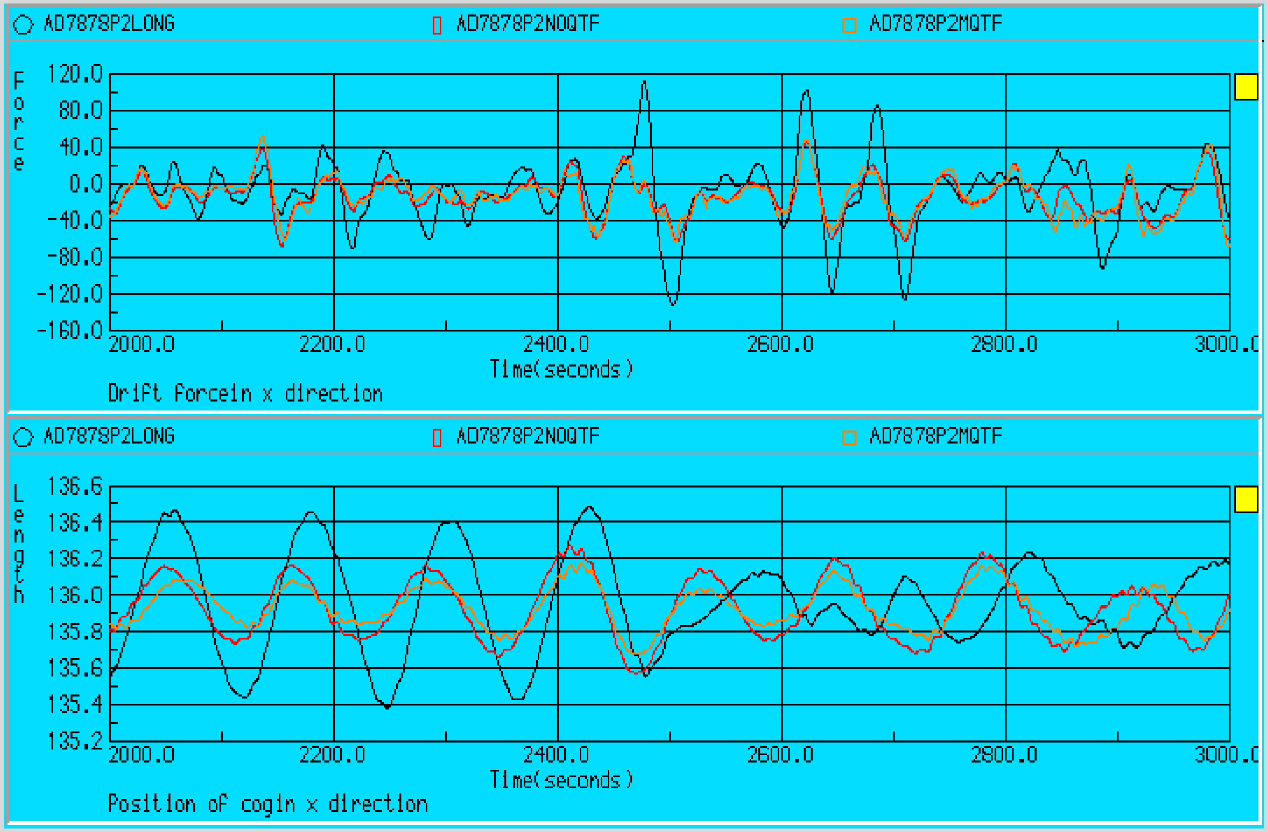
Input data



Iterative procedure

Time domain analysis (Aqwa-Drift)

- **5% CPU increment for 7-sub-direction coupling drift force calculation**
- **Evident differences between 3 treatment results**



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Frequency domain analysis (Aqwa-Fer 14.5)

- **Output total force/response spectrum contributed by all sub-spectra**
- **Output RAO et al in main sub-direction (with max. Hs)**
- **Optionally output RAO in specified sub-direction (SSPC in deck18)**

Comparison of the significant translational motions

	Surge (m)		Sway (m)		Heave (m)	
	Drift freq.	Wave freq.	Drift freq.	Wave freq.	Drift freq.	Wave freq.
Long crested	1.073	0.018	0.000	0.000	0.011	0.036
Spread, no coupling	0.558	0.025	0.563	0.040	0.007	0.055
Spread, coupling	0.620	0.025	0.618	0.040	0.007	0.055

6. Conclusions

- Spectral group is introduced to model multi-directional waves
- The directional coupling mean QTFs are calculated and used in Aqwa
- Extended Newman's approximation provides a fast & relatively accurate approach with acceptable hard disk/memory requirements
- Multiple directional waves and directional coupling QTF should be considered for moored offshore structure hydrodynamic analysis.

Thanks!

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